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EE 272 - Dynamics of Lasers
Homework 2 : Semiconductor lasers
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We consider a distributed feedback (DFB) semiconductor laser. The cavity length is 300-μm long. The width and thickness of the active area is 1.2-μm and 0.2-μm respectively. The internal loss is given by the coefficient α_i=40 cm^{-1}, and the reflectivity (in intensity) of each facet is $R_1 = R_2 = 0.3$. The semiconductor laser rate equations describing the dynamics of carrier and photon numbers within the laser cavity are given by :

$$\frac{dS}{dt} = \left[G_N(N - N_t) - \frac{1}{\tau_p}\right]S \tag{1}$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_s} - G_N(N - N_t)S \tag{2}$$

with S the photon number, N the carrier number, J the pump current, τ_p the photon lifetime, τ_s the carrier lifetime and e =1.6 $\times$ 10^{-19}C the elementary charge of the electron. We assume a linear gain dependence such as $G = G_N(N - N_t)$ with G_N=1.4 $\times$ 10^4 s^{-1} a temporal coefficient of dynamic gain and N_t =7.2 $\times$ 10^7 the carrier number at the optical transparency. The group index is n_g=4. The speed of light is c=3 $\times$ 10^8 m/s.

Question 1 As opposed to what we saw in class, the confinement factor is not included into the rate equations. Why is that ?

When using carrier and photon numbers, the confinement factor is not needed into the rate equations. Indeed, by setting the time rate of change of the carriers and photons equal to the sum of rates into, minus the sum of rates out of the respective reservoirs (photons and electrons), we arrive at the carrier and photon number rate equations. Then dividing out the volumes, and using $\Gamma = V/V_p$, (with V_p the cavity volume occupied by the photons which is much larger than the active region volume occupied V by electrons), we obtain the density rate equations. However, it is important to note that the gain term in the density version has a Γ in the photon density rate equation, but not in the carrier density rate equation. While in the number rate equations, the gain term is symmetric. This asymmetry in the density rate equations is often overlooked in the literature. A complete demonstration can be found in the textbook from *L. A. Coldren and S. W. Corzine, Diode lasers and Photonic Integrated Circuits, Chapter 5, Wiley, 1995*.

Question 2 Calculate the photon lifetime τ_p.

$\tau_p = [c/n \times (\alpha_i + \alpha_m)]^{-1} = [c/n \times (\alpha_i + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)]^{-1}$. Taking $\alpha_m \approx 40$ cm^{-1} we find $\tau_p \approx$1,7 ps.

Question 3 Calculate the carrier number at threshold N_{th}.

Using the threshold condition (gain=loss) we find $N_{th} = N_t + 1/G_N\tau_p \approx 1.1 \times 10^8$.

Question 4 Calculate the threshold current J_{th} for τ_s=1 ns.

Using the steady-state conditions we show $J_{th} = eN_{th}/\tau_s \approx 18$ mA.

Question 5 Express the photon number S_0 as a function of the pump current. Calculate S_0 for J/J_{th}=2.
From $J/e - N_{th}/\tau_s - G_N(N_{th} - N_t)S_0 = 0$ and given that $J_{th} = eN_{th}/\tau_s$ we find $S_0 = \frac{\tau_p}{e}(J - J_{th}) \approx 2 \times 10^5$.

Question 6 Applying a small-signal analysis on Eqs. (1)-(2) allows retrieving the so-called relaxation oscillation frequency (ROF) of the laser such as :

$$f_R = \frac{1}{2\pi}\sqrt{\frac{G_N S_0}{\tau_P}} \tag{3}$$

Using the value of S_0 for J/J_{th}=2, calculate the ROF in GHz. Give possible ways to increase the ROF.
We find $f_R \approx$ 6,4 GHz. To increase the ROF possible directions are :
(1) Biasing the laser at a higher pump current J which gives a higher photon number S_0 ;
(2) Reducing the photon lifetime by decreasing the cavity length and/or by changing the transmission mirrors ;
(3) Increasing the differential gain coefficient by cooling the device, using doping active areas or quantum confined semiconductors.

Question 7 Assuming $G_N N_t \tau_p << 1$, show that the ROF can be expressed as :

$$f_R = \frac{1}{2\pi}\sqrt{\frac{1}{\tau_s \tau_p}\left[\frac{J}{J_{th}} - 1\right]} \tag{4}$$

Taking J/J_{th}=2, calculate the ROF in GHz. Conclusions.
From $J/e - N_{th}/\tau_s - G_N(N_{th} - N_t)S_0 = 0$ and assuming $N_{th} > N_t$ we get, $G_N S_0 = [J/e - N_{th}/\tau_s]$. Injecting this last expression into (3) leads to (4). Here we find $f_R \approx$ 4 GHz, meaning that the assumption is not verified. The latter implies that a smaller number of carriers is required to reach the optical transparency.